

Weak Interactions

- We have discussed in some detail
Electromagnetic Interactions
Mediated by Spin-1 photons
- Strong Interactions
Mediated by Spin-1 gluons
- We talked about QED and QCD, the coupling strength α_e and α_s , the coupling constants and their running with q^2 (or Q^2).
- We discussed form factor of the proton and parton structure functions
- Now to Weak Interactions
 - Also a fundamental force mediated by exchange of Spin-1 W^\pm and Z bosons, between quarks and leptons.
 - Has coupling denoted by g_W or α_W
 - W^\pm and Z bosons are massive

$$M_W \approx 80 \text{ GeV}$$

$$M_Z \approx 91 \text{ GeV}$$

Discovered at
CERN in
1983 by
Rubbia et al.,

→ ∴ The range of weak force

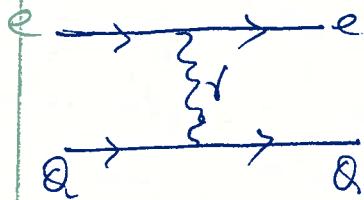
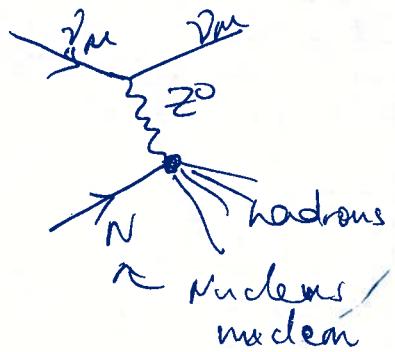
$$R_{W,Z} = \frac{\hbar}{M_W c} \simeq 2 \times 10^{-18} \text{ m} = \underline{\underline{2 \times 10^{-3} \text{ fm}}}$$

At low energies, this range is very small compared to the de Broglie wavelengths of particles involved and so the interaction can be approximated as zero-range. Not so at high energy.

Massive charged bosons as mediators of weak interactions proposed by Klein in 1938 (W^\pm)

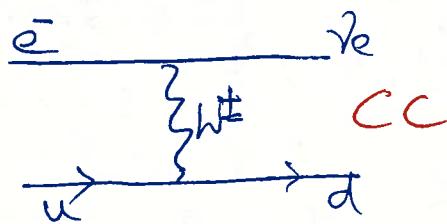
Glashow, Salam and Weinberg's Electroweak theory in 1960's predicted Z^0 in addition to W^\pm

Neutral currents (i.e., weak interaction with exchange of Z^0) experimentally observed in 1983 at CERN

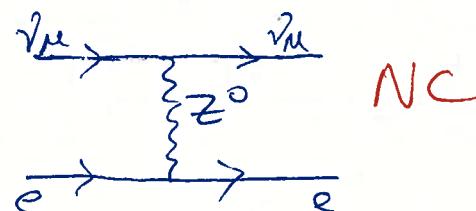


EM interaction

Interaction between two centered electric currents j_{em}



Charged current (CC)
weak interaction



Neutral current (NC)
weak interaction

Interaction between weak currents j_{weak}

In CC, electric charge of each weak current changes

In NC, electric charge does not change.

Examples of charged current (cc) weak decays:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \bar{\nu}_\mu$$

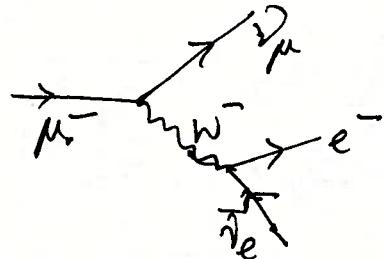
Purely leptonic

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Semi-leptonic

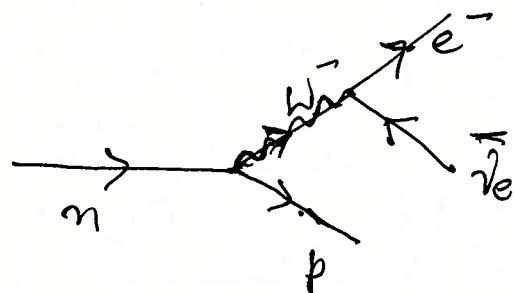
$$\Lambda \rightarrow p + \pi^-$$

purely hadronic
or non-leptonic



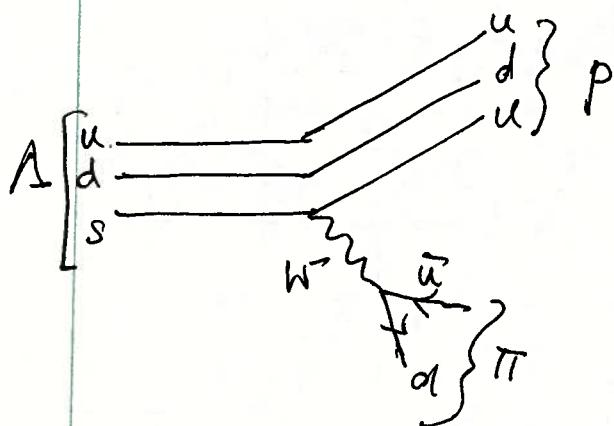
Beta decay of Muon

$\tau =$



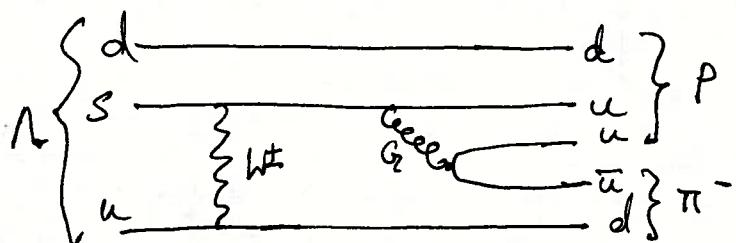
Beta decay of neutron

$\tau =$



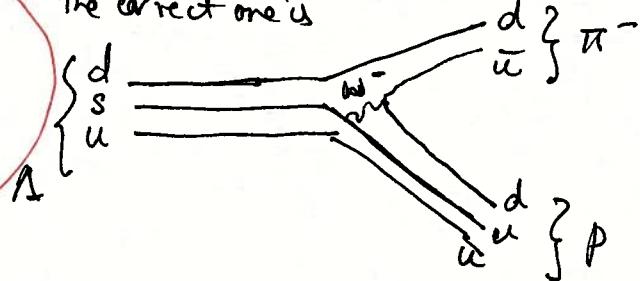
$\Lambda \rightarrow p + \pi^-$
hyperon

higher order
with gluon emission
too



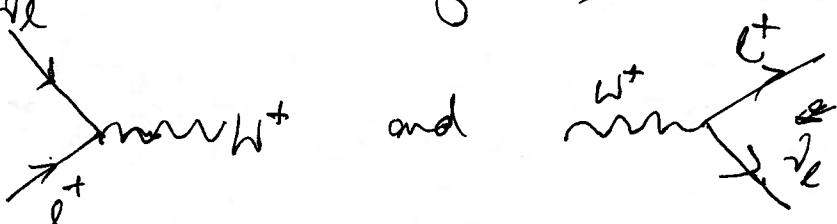
The fig 8.9(5) in
M&S p.225
is wrong

The correct one is



(3)

As in electromagnetic interactions, there are
^(+ eight) eight basic vertex processes involving $l^\pm, \bar{e}/\bar{\nu}_e$ and W^\pm .
 But unlike in case of EM,



+ similar ones for W^-

can occur in free space since

$$\cancel{M}_W > M_l + M_{\bar{\nu}_e}$$

and no energy and momentum can both be conserved

$W^\pm \rightarrow l \bar{v}_l$ ($\bar{l} v_e$) are the dominant mechanism
 for leptonic decays. (used in the discovery

of W^\pm . Z discovered via $Z^0 \rightarrow l^+ l^-$)

$$W^+ \rightarrow e^+ \bar{\nu}_e$$

$$W^- \rightarrow e^- \bar{\nu}_e$$

$$W^+ \rightarrow \mu^+ \bar{\nu}_\mu$$

$$W^- \rightarrow \mu^- \bar{\nu}_\mu$$

$$W^+ \rightarrow \tau^+ \bar{\nu}_\tau$$

$$W^- \rightarrow \tau^- \bar{\nu}_\tau$$

So one simply writes $W \rightarrow e \bar{\nu}_e$, $W \rightarrow \mu \bar{\nu}_\mu$, $W \rightarrow \tau \bar{\nu}_\tau$

We also have $W \rightarrow u \bar{d}$, $c \bar{s}$

For these $\times 3$ $\times 3$ for color.

$\therefore W^\pm$ decay has 9 possible final states.

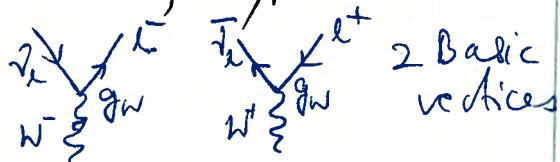
$\therefore W^+ \rightarrow e^+ \bar{\nu}_e$ therefore has branching fraction
 of $\underline{\frac{1}{9}}$

Lepton Universality

Each of the lepton couples to the W boson with the same coupling strength g_W . (or α_W) $\alpha_W = \frac{g_W^2}{4\pi}$

In other words, the coupling strength is independent of the lepton type i.e., e, μ or τ . ^{Coupling constant}

$$g_e \approx g_\mu \approx g_\tau \equiv g_W$$



This is verified by looking at decays of

$$\mu \rightarrow e \bar{\nu}_e \nu_\mu \text{ and } \tau \rightarrow \mu \bar{\nu}_\mu \bar{\nu}_\tau$$

$$\frac{g_\tau/g_\mu}{g_e/g_\mu} = 0.999 \pm 0.003; \quad \frac{g_\tau/g_\mu}{g_e/g_\mu} = 1.001 \pm 0.004 \quad \left\{ \begin{array}{l} \text{using } \Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) \propto \Lambda^2 m_\mu^5 \\ \text{etc.} \end{array} \right.$$

$$g_\mu/g_e = 1.001 \pm 0.002$$

Similarly the partial width of the Z boson decays to the three leptonic modes (measured at LEP) are the same

$$Z \rightarrow e^+ e^- : \mu^+ \mu^- : \tau^+ \tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005$$

Also, $\Gamma(W \rightarrow e \gamma) \approx 0.230 \pm 0.008 \text{ GeV}$ (measured)
From dimensional argument,

$$\Gamma(W \rightarrow e \gamma) \approx \alpha_W M_W \approx 80 \text{ GeV}$$

$$\therefore \alpha_W \approx \frac{1}{350}$$

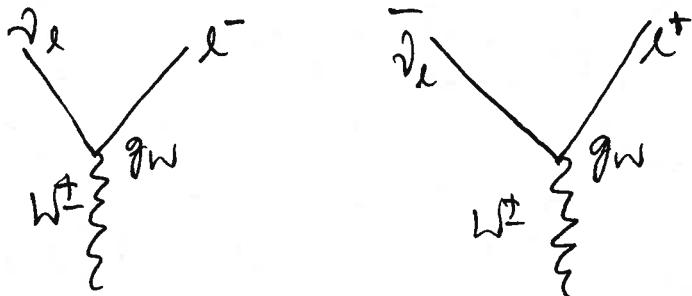
A more detailed calculation $\Rightarrow \Gamma(W \rightarrow e \gamma) = 2 \alpha_W M_W / 3$

$$\boxed{\alpha_W = 0.0043 \pm 0.0002}$$

In contrast to the universality of the lepton couplings, the couplings of the quarks to the weak bosons do depend on the quark flavors involved.

Quark Mixing

The basic vertices for W^\pm -lepton interactions are



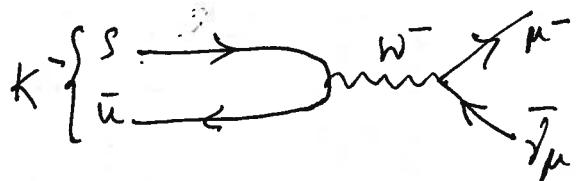
for the time being, if we assume just the first two generations,

$(\bar{e} e)$, $(\bar{\nu}_e \nu_e)$ have identical weak interaction

Will the first two generations of quarks have similar weak interactions?

i.e., is $g_{ud} = g_{cs} = g_W$?

If this is the case, then



would be forbidden
because W would not
couple across generations.

But, these weak decay $K \rightarrow \pi \nu \bar{\nu}$ is experimentally observed and naturally explained by the above diagram. This can be incorporated if we allow mixing between d and s quarks.

Quark Mixing (contd.)

i.e.,

$$d' = d \cos \theta_c + s \sin \theta_c$$

and

$$s' = -d \sin \theta_c + s \cos \theta_c$$

where θ_c is the Cabibbo angle.

So the quark doublets would then be written as

$$\begin{pmatrix} u \\ d' \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s' \end{pmatrix}$$

Then the couplings for the W^\pm -quark interactions would become

$$g_{ud} = g_{cs} = g_W \cos \theta_c$$

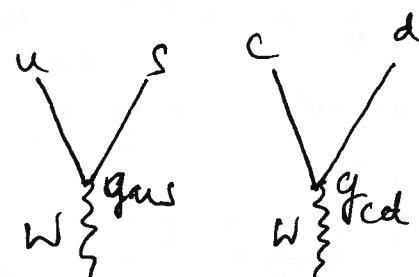
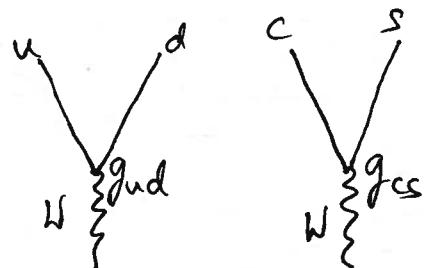
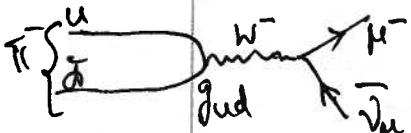
and one gets new couplings that are allowed

$$g_{us} = -g_{cd} = g_W \sin \theta_c$$

Since g_W is known already known W the W -quark couplings can be deduced in terms of θ_c .

g_{ud} , g_{us} etc. can be determined from hadron decays.

for example
$$\frac{\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2 \theta_c$$



(7)

Experimentally,

$$\tan \theta_c = 0.232 \pm 0.002$$

$$\Rightarrow \theta_c = 13.1 \pm 0.1 \text{ degrees.}$$

One can cross-check these values using rates of other decays.

for example, $n \rightarrow p + e^- + \bar{\nu}_e$

has a g_{ud} vertex coupling.

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \gamma_\mu$$

has g_W coupling

\therefore the ratio of these decay rates

$$\Rightarrow \left(\frac{g_{ud}}{g_W} \right)^2 = \cos^2 \theta_c$$

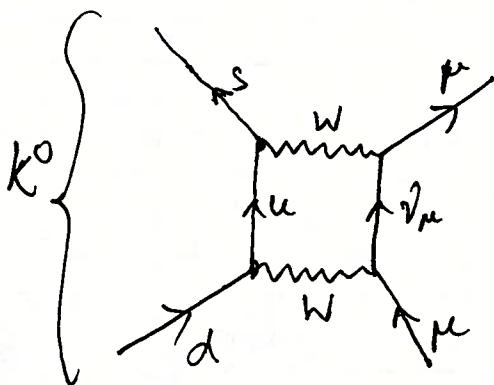
Also note that

$$\frac{g_{us}^2}{g_{ud}^2} = \frac{g_{cd}^2}{g_{cs}^2} = \tan^2 \theta_c = 1/20$$

and so decays involving g_{us} and g_{cd} couplings (or $u\bar{u}W$ and $c\bar{c}W$ vertices) are "Cabibbo-suppressed" by this factor ~ 20 relative to the "Cabibbo-allowed" decays.

GIM Mechanism

The $K^0 \rightarrow \mu^+ \mu^-$ is allowed through the weak interaction as follows



has udW and usW vertices on the left and leptonic vertices in the final state.

$$\therefore \text{Decay rate} \propto \sin\theta_c \cos\theta_c$$

For $K_0^0 \rightarrow \mu\mu < 3 \times 10^{-7}$ sec.

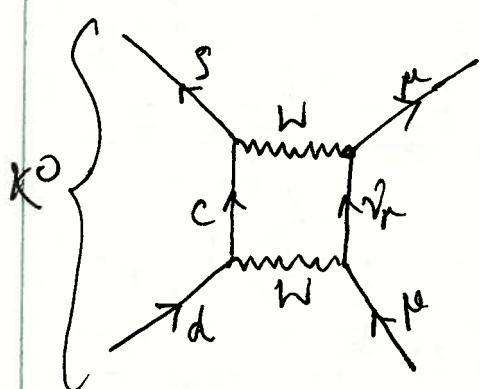
This is far greater than calculated rate.

This dilemma was solved by a proposal in 1970

by Glashow, Iliopoulos and Maiani (GIM)

dubbed GIM Mechanism.

They introduced the fourth quark "Charm" and the following diagram



Note "c" replaces "s" in the left internal line.

This diagram also contributes to $K^0 \rightarrow \mu^+ \mu^-$ and has an amplitude proportional to

$$-\sin\theta_c \cos\theta_c$$

Hence the small decay rate! So this diagram cancels the top one! Not completely since m_c is different from m_s !

The Third Generation

In 1971, four leptons $\nu_e, e^-; \nu_\mu, \mu^-$ and three quarks u, d, s were known. Glashow, Dimopoulos and Maiani proposed a fourth quark c to provide lepton-quark symmetry and to solve problems associated with neutral currents.

In 1974 there was the "November Revolution" - the discovery of the charm quark.

But c lepton was discovered in 1975 to hint 3rd generation and then b quark in 1977 to confirm the 3rd generation.

Finally, the top quark in 1995 and ν_τ in 2000 completed the 3 doublet, 3 generation structure of the leptons and quarks.

Quark Mixing with the 3 Generations on hand

So we had Cabibbo mixing, which can be written as

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

\curvearrowleft "Cabibbo-rotated" states

Now, we have $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$ for quarks.

$$\Rightarrow \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo
Kobayashi
Maskawa matrix

\leftarrow CKM Matrix

||

$V_{\alpha\beta}$ ($\alpha = u, c, t$; $\beta = d, s, b$)

And, as before,

$$g_{\alpha\beta} = g_W V_{\alpha\beta}$$

So, we have, in the generalized scheme with three generations of quarks and leptons,

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

"weak interaction generations"

with the CKM matrix (relating to the "physical states" of quarks)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{\text{CKM}} = \begin{pmatrix} 0.9738 & 0.2272 & 0.0040 \\ 0.2271 & 0.9730 & 0.0422 \\ 0.0081 & 0.0416 & 0.9991 \end{pmatrix}$$

using experimental observations

Note the diagonal elements are ≈ 1

and the couplings of the 3rd generation with 1st and 2nd are very small

\Rightarrow Cabibbo scheme with 2 generations is a reasonable approximation.

If the mixing of b with (d, s) can be neglected,
then,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

then $b' = b$ and we get back to the Cabibbo matrix relation.

This seems to be a pretty good approximation

and $|V_{ub}|^2 \approx 2 \times 10^{-5}$; $|V_{cb}|^2 \approx 2 \times 10^{-3}$

from experimental study of b hadron decays.

Take $B \rightarrow D \ell \bar{\nu}$

$$\Gamma(b \rightarrow c \ell \bar{\nu}) = \frac{R(b \rightarrow c \ell \bar{\nu})}{\tau_B}$$

$$= \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 f$$

\uparrow Correction factor
for bound "b" quark

$$\Rightarrow V_{cb} = 0.040 \pm 0.005$$

In fact, also, the matrix elements are not all independent.

One has three Euler angles and one phase angle or 3 "generalized Cabibbo" angles and a phase.

$$V_{cb} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos \theta_{ij}$; $s_{ij} = \sin \theta_{ij}$ and δ is the ^{CPM-} phase

The phase angle enters the wavefunction as $e^{i(\omega t + \delta)}$ which can violate time reversal or CP violation.

The matrix V_{cb} also has to be unitary.

$$\text{i.e., } V^T V = I$$

This leads to unitarity constraints and unitarity triangle.

$$\sum_i V_{ij} V_{ik}^* = \delta_{jk} ; \sum_j V_{ij} V_{kj}^* = \delta_{ik}$$

$$\therefore |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \text{ etc.}$$

$$\text{and } V_{ud}V_{us}^* + V_{ud}V_{ub}^* + V_{us}V_{ub}^* = 0, \text{ etc.}$$

The Top Quark

$$m_t \simeq 173 \text{ GeV}$$

Much heavier than the W^\pm bosons

$$\therefore t \rightarrow q + W \quad (\text{weak decay})$$

$(q = d, s, b)$

with rates proportional to $|g_{tq}|^2$

But, in the approximation we made,

$$g_{td} = 0, g_{ts} = 0 \text{ and } g_{tb} = g_W$$

Then, the only significant decay mode is,

$$t \rightarrow W + b$$

with decay width $\Gamma \sim \alpha_W M_W \sim 1 \text{ GeV}$
and $\tau \sim 10^{-24} \text{ s}$

A hadron state cannot form in a time less
(of size $\sim 1 \text{ fm}$)

$$\text{than } t \approx d/c = 0(10^{-23} \text{ s}).$$

The other quarks have lifetimes $\gtrsim 10^{-12} \text{ s}$ and do
have plenty of time to form hadrons, while
the top quark does not.

I have already covered top production, decay and measurements at the Tevatron.

For more on the Discovery of the Top quark and top quark physics, see

P. C. Bhat, et.al.,

International Journal of Modern Physics,
Vol. 13, No. 30 (1998) 5113-5218.

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